Shaft CenterLINES

Half-Power Bandwidth method for the evaluation of synchronous and nonsynchronous quadrature stiffnesses

Nomenclature

A, α	Rotor synchronous response amplitude and phase, respectively
A_{res}	Amplitude A at resonance
B, β	Rotor amplitude and phase in response to nonsynchronous excitation
C, σ	Rotor 2x response amplitude and phase, respectively
D, K, M	Rotor first lateral mode modal damping, stiffness and mass, respectively
e	$= 2.7183$
E, Θ	Cracked rotor synchronous response vector components
F, γ	Nonsynchronous perturbation force amplitude and phase, respectively
j	$= \sqrt{-1}$
m, r, δ	Rotor modal unbalance mass, radius, and angular orientation
P, ϵ	Constant radial force amplitude and angular orientation
Q_{nsyn}	Nonsynchronous amplification factor
Q_{syn}	Synchronous amplification factor
$Q_{syn\Delta}$	Synchronous amplification factor for a cracked shaft
Q_{2x}	Amplification factor for 2x response component
Δ	Difference of rotor stiffness in two orthogonal directions perpendicular to rotor axis, a measure of a rotor crack
ζ	Modal damping factor
λ	Fluid average circumferential velocity ratio
ω	External perturbation frequency
ω_{res}	Resonance frequency
$\omega_{45°}, \omega_{135°}$	Frequencies at the rotor response phase lag of 45° and 135°, respectively
Ω	Rotative speed
$\Omega_{res} \approx \sqrt{K/M}$	First balance resonance speed
$\Omega_{res\Delta}$	Rotative speed at 1X resonance of an anisotropic rotor
Ω_{res2}	Rotative speed at resonance of 2X component
$\Omega_{45°}, \Omega_{135°}$	Rotative speeds at the 1X response phase lag of 45° and 135°, respectively.

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This article discusses the use of the Half-Power Bandwidth (HPB) method for the evaluation of the synchronous and nonsynchronous quadrature dynamic stiffnesses and amplification factors for rotor/bearing/seal systems. Traditionally, the HPB method is referred to as the modal damping identification method. This article shows that this method not only provides damping, but also the entire quadrature dynamic stiffness, which, in the case of rotor lateral modes, also contains tangential, cross-coupled stiffness terms. The HPB method, when used on rotor nonsynchronous perturbation test data, yields an evaluation of the destabilizing, flow-induced forces resulting from the fluid environment. When the HPB method is used for rotor 2X response, it may provide information of an impending shaft crack.

Synchronous response of an isotropic rotor in the range of its first lateral mode

As has been shown, [References 2-5], the synchronous response vector, due to unbalance of a simple isotropic rotor, for either vertical or horizontal motion in the range of the rotor first lateral mode, is as follows:

$$Ae^{j\alpha} = \frac{mr\Omega^2 e^{j\delta}}{K - M\Omega^2 + j\Omega D(1-\lambda)}, \quad j = \sqrt{-1} \tag{1}$$

where $Ae^{j\alpha}$ is the 1X response vector (A = amplitude, α = phase), K, M, D are modal stiffness, mass, and damping; m, r, δ►

are modal unbalance mass ("heavy spot"), radius, and angular orientation, respectively, Ω is the rotative speed and λ is the fluid average circumferential velocity ratio, the cross-coupled tangential effect of the fluid surrounding the rotor [2]. The product $D(1-\lambda)$, is referred to as "modal effective damping."

In Equation (1) the expression in the denominator represents the system complex dynamic stiffness with its direct term, $K-M\Omega^2$, and quadrature term, $\Omega D(1-\lambda)$.

In the "real" (direct) or "imaginary" (quadrature) plane, the response vector (1) represents a point for any constant rotative speed, Ω. For the speeds varying from zero to some value higher than the resonant one, the response vector draws a figure similar to a circle (Figure 1), known as a modal circle, "Nyquist circle," or more commonly, a "polar plot."

There are several characteristic points on the polar plot. At $\Omega=0$, there is $A = 0$ and $\alpha=\delta$; thus, the polar plot begins at the coordinate origin with the response vector's phase, δ, the same as the heavy spot phase. For low values of damping, at $\Omega=\Omega_{res}\cong\sqrt{K/M}$, the response vector reaches its highest, resonant value, and the phase lags the original value δ (at $\Omega=0$), by 90 degrees. At high rotative speed, the inertia term, $M\Omega^2$, in Equation(1) becomes dominant, and the response vector amplitude tends to a constant value, while the phase lags 180 degrees from its value at $\Omega = 0$.

There are two more characteristic points on the polar plot, namely at $\alpha=\delta-45^\circ$ and $\alpha=\delta-135^\circ$. These points are situated on the polar plot circle diameter which is perpendicular to the response vector at resonance (Figure 1).

The values of the rotative speed at these points, as well as at resonance, leads to the basic equation of the half-power bandwidth method:

$$\frac{\Omega_{135^\circ}-\Omega_{45^\circ}}{\Omega_{res}}=\frac{D(1-\lambda)}{\sqrt{KM}}=2\zeta(1-\lambda) \tag{2}$$

where $\zeta=D/(2\sqrt{KM})$ is known as a "damping factor." The inverse of expression (2) represents the basic equation for the synchronous amplification factor Q_{syn}:

$$\frac{\Omega_{res}}{\Omega_{135^\circ}-\Omega_{45^\circ}}=Q_{syn}=\frac{1}{2\zeta(1-\lambda)} \tag{3}$$

Thus, in order to evaluate the rotor first mode effective damping and corresponding synchronous amplification factor, three speed values from the polar plot are required: the resonant speed at the highest value of the response amplitude and two speeds at the polar plot circle diameter perpendicular to the response vector at resonance (Figure 1). The calculation is performed using Equations (2) or (3).

Trending the synchronous amplification factor may provide valuable information about possible changes in the damping factor or in fluid-induced forces, represented by the ratio λ, which, if increased to one, may lead to rotor instability. Note that the synchronous amplification factor does not depend on the exciting unbalance force, provided that the response amplitude stays within a linear response range.

The amplification factor can also be estimated from the 1X response Bode plot as a ratio of the response amplitude at resonance to the amplitude at high rotative speed (lower, however, than the second mode frequency range). This evaluation is, however, less accurate.

Short history of "Half-Power Bandwidth"

The name "Half-Power Bandwidth" originated from the response vector voltage (V) of an electrical circuit, which is usually plotted in logarithmic scale. The response amplitudes at $\alpha=\delta-45^\circ$ and $\alpha=\delta-135^\circ$ have approximately the same value, which is a $1/\sqrt{2}$ fraction of the resonant amplitude:

$$A_{45^\circ}\approx A_{135^\circ}\approx\frac{V_{res}}{\sqrt{2}}\quad\text{(electrical)}$$

$$\approx\frac{mr}{\sqrt{2}\,D(1-\lambda)}\sqrt{\frac{K}{M}}=\frac{A_{res}}{\sqrt{2}}\quad\text{(mechanical)}$$

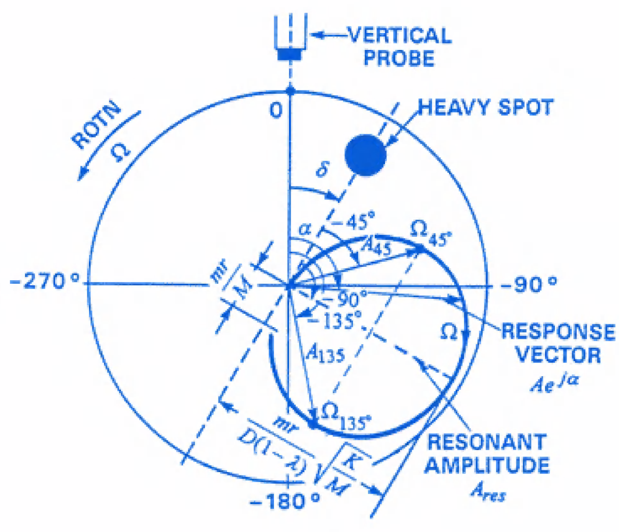

Figure 1
Polar plot of the synchronous (1X) response of a rotor, measured by a lateral probe.

Since power is proportional to the square of the voltage, the $A_{45°}$ and $A_{135°}$ points also represent the half power points of voltage.

The voltage or displacement amplitude ratio $1/\sqrt{2}$ corresponds to approximately a 70% or 3dB amplitude reduction. Thus, the points defined by that ratio are sometimes called the 70% or 3dB amplitude points, as well as the half-power points. These points define the 3db amplitude bandwidth used to calculate Q_{syn}.

With current data acquisition technology, the points can be determined most accurately from the numerical listing of the data. The second choice for accurate data would be the polar plot, which typically offers better resolution of the phase and frequency data than the Bode plot.

Nonsynchronous response of an isotropic rotor with fluid interaction in the range of its first lateral mode

Nonsynchronous excitation of a rotor occurs when an external-perturbation, rotating force with frequency ω ($\omega \neq \Omega$) is applied, while the rotor rotates at a constant rotative speed, Ω.

The response vector, $Be^{j\beta}$, within the first lateral mode of a very simple isotropic rotor excited by the nonsynchronous rotating force, is as follows [3]:

$$Be^{j\beta} = \frac{Fe^{j\gamma}}{K - M\omega^2 + jD(\omega - \lambda\Omega)} \tag{4}$$

where F and γ are exciting force amplitude and angular orientation, respectively. Now the cross-coupled, tangential effect of the fluid surrounding the rotor appears as a separate component, $D\lambda\Omega$, of the dynamic stiffness.

A resonance ("direct" or "mechanical") still exists at $\omega_{res} = \sqrt{K/M}$. The modification occurs in the quadrature dynamic stiffness term. This term in Equation (4) can no longer be always positive, as it was in Equation (1). Due to the fluid interaction transferring rotative energy into the lateral vibration mode through the cross coupling stiffness term, $D\lambda\Omega$, the entire quadrature dynamic stiffness may become zero. This leads to the quadrature resonance conditions at $\omega_{qres} = \lambda\Omega$ in a simple rotor system [3,4].

If the Half-Power Bandwidth analysis is applied to the rotor excited with nonsynchronous sweep frequency ω, one can easily deduce that the nonsynchronous amplification factor for the direct resonance $\omega_{res} = \sqrt{K/M}$ is as follows:

$$Q_{nsyn} = \frac{\omega_{res}}{\omega_{135°} - \omega_{45°}} \approx \frac{K}{D(\sqrt{K/M} - \lambda\Omega/\sqrt{1+\zeta^2})} = \frac{Q_{syn}(1-\lambda)}{1-(\lambda\Omega\sqrt{M/K}/\sqrt{1+\zeta^2})} \tag{5}$$

The half-power points correspond to the phases γ–45° and γ–135°. The nonsynchronous amplification factor, Q_{nsyn}, is magnified in comparison to Q_{syn} (Equation (3)), by the factor which depends on the closeness of the quadrature resonant frequency, $\omega_{qres} = \lambda\Omega$, to the direct resonant frequency, $\omega_{res} = \sqrt{K/M}$. If both resonant frequencies coincide, that is, if $\lambda\Omega = \sqrt{K(1+\zeta^2)/M}$, then, obviously, an instability occurs, and the response amplitude B, as well as Q_{nsyn}, infinitely increase.

From Equation (5), it is clearly seen that the Half-Power Bandwidth method applied to rotor responses provides more than a simple damping measure. It provides the measure of the entire quadrature dynamic stiffness, containing damping and the destabilizing, cross-coupled stiffness generated by the rotor rotational mode. For more complex systems, more terms may appear in the equation; however, the fundamental rules remain essentially the same.

1X amplification factor of a cracked shaft

The synchronous 1X response amplitude, E, and phase Θ, of a transversely-cracked rotor excited by an unbalance force, can be presented as follows [5]:

$$E = mr\Omega^2 \frac{\sqrt{[(K-M\Omega^2)\cos\delta + D\Omega\sin\delta]^2 + [(K-M\Omega^2)\sin\delta - D\Omega\cos\delta]^2}}{(K-M\Omega^2)(K-\Delta-M\Omega^2) + D^2\Omega^2} \tag{6}$$

$$\Theta = \arctan\frac{(K-M\Omega^2)\sin\delta - D\Omega\cos\delta}{(K-\Delta-M\Omega^2)\cos\delta + D\Omega\sin\delta} \tag{7}$$

where Δ is the difference between the rotor stiffnesses in two orthogonal directions perpendicular to the rotor axis. The parameter Δ can represent a measure of a shaft transversal crack. For clarity of presentation, the factor, λ, has been omitted in Equations (6) and (7). As can be seen from Equations (6) and (7), the cracked-shaft synchronous response depends on the location of the crack in relation to the heavy spot. The worst case results if they are separated by 45 degrees, that is if $\delta = 45°$. This is the case that will be discussed in this article.

The 1X resonance occurs when:

$$\Omega_{res\Delta} = \sqrt{\frac{K-\Delta/2}{M}} \tag{8}$$

which, for an impending crack, is slightly lower than the previously discussed resonant frequency, $\sqrt{K/M}$.

Using the Half-Power Bandwidth method, the synchronous amplification factor can be evaluated. Since at $\Omega = 0$ the►

assumed phase is $\Theta = \delta = 45°$, the half-power frequency points to be calculated occur at $\Theta = 0°$ and $\Theta = -90°$, which are $-45°$ and $-135°$ from the origin.

Using the Half-Power Bandwidth method, the synchronous amplification factor of a cracked rotor is as follows:

$$Q_{syn\Delta} = \frac{\Omega_{res\Delta}}{\Omega_{135°} - \Omega_{45°}} \approx \frac{\sqrt{1 - \dfrac{\Delta}{2K}}}{2\zeta - \dfrac{\Delta}{2K}} = \frac{\sqrt{1 - \dfrac{\Delta}{2K}}}{\dfrac{1}{Q_{syn}} - \dfrac{\Delta}{2K}} \qquad (9)$$

With an increasing value of Δ for an impending crack in the rotor, the amplification factor increases significantly. Its growth rate depends on the amount of damping in the system. The "crack ratio," $\Delta/2K$, appears in the amplification factor as another "tangential component" subtracted from the damping factor (ζ), thus decreasing the denominator of the amplification factor (9). It is necessary to point out that the growth of the amplification factor depends considerably on the crack versus unbalance location. Equation (9) defines the worst condition only. In any case, trending the synchronous amplification factor may give information on an impending rotor crack. Note that, as before, the amplification factor does not depend on the unbalance force amplitude.

2X amplification factor of a cracked shaft

The 2X lateral response of the rotor may occur as the second harmonic of high 1X vibrations, due to nonlinearities in the system. The 2X response may, however, be entirely independent from 1X vibrations. It may be generated by a constant radial force applied to an anisotropic rotor. The rotor anisotropy may result from a rotor transversal crack. For the rotative speed, Ω, the rotor 2X response vector, $Ce^{j\sigma}$, has, in this case, the following amplitude, C, and phase, σ [5]:

$$C = \frac{P\Delta}{2(2K-\Delta)\sqrt{\left(\dfrac{K(K-\Delta)}{2K-\Delta} - 2M\Omega^2\right)^2 + D^2\Omega^2}} \qquad (10)$$

$$\sigma = -\epsilon - \arctan \frac{D\Omega}{\dfrac{K(K-\Delta)}{2K-\Delta} - 2M\Omega^2} \qquad (11)$$

where Δ is a measure of a shaft crack, and P and ϵ are constant radial force amplitude and angular orientation, respectively. The force, P, may result from rotor misalignment, from fluid flow side-load (as in single volute pumps) or from gravity (as on horizontal rotors). For clarity of presentation, the ratio λ has been omitted in Equations (10) and (11).

The 2X polar plot starts at $\Omega = 0$, with the phase $\sigma_o = -\epsilon$. At resonance, i.e. when $\Omega = \Omega_{res2} \approx \frac{1}{2}\sqrt{K/M}$, the 2X response amplitude is the highest: $C_{res} = PM\Delta/DK(K-\Delta)$, with the phase $\sigma_{res} = -\epsilon - 90°$. At high rotative speeds, the amplitude approaches zero, with the phase approaching $\sigma_{\infty} = -\epsilon - 180°$. The 2X polar plot behaves, therefore, very similarly to the 1X polar plot circle.

The application of the Half-Power Bandwidth method results in:

$$\frac{\Omega_{135°} - \Omega_{45°}}{\Omega_{res2}} = \frac{D}{\sqrt{2M\dfrac{K(K-\Delta)}{2K-\Delta}}} = \zeta\sqrt{\frac{2(2K-\Delta)}{K-\Delta}} \qquad (12)$$

Therefore, the amplification factor, Q_{2x}, for the 2X response is:

$$Q_{2x} = \frac{\Omega_{res2}}{\Omega_{135°} - \Omega_{45°}} = \frac{1}{\zeta}\sqrt{\frac{K-\Delta}{2(2K-\Delta)}} = Q_{syn}\sqrt{\frac{1 - \dfrac{\Delta}{K}}{1 - \dfrac{\Delta}{2K}}} \qquad (13)$$

This 2X amplification factor includes the effect of the rotor anisotropy, such as is generated by a shaft crack. For the isotropic rotor (when $\Delta = 0$), there is $Q_{2x} = 1/2\zeta$, which means $Q_{2x} = Q_{syn}$ (with $\lambda = 0$). If, for example, $\Delta = 0.3K$, then $Q_{2x} = 0.91Q_{syn}$. If $\Delta = 0.5K$, then $Q_{2x} = 0.82Q_{syn}$. While the 2X response amplitude (10) increases almost proportionally to the crack parameter Δ, surprisingly, the 2X amplification factor does not increase, but decreases with Δ. It is also substantially lower than $Q_{syn\Delta}$. Trending the 2X amplification factor may, however, provide valuable information on an impending shaft crack.

Conclusion

The Half-Power Bandwidth method evaluates quadrature stiffness components. These may consist of destabilizing, fluid-induced, as well as shaft-crack-related, terms. Measurement and trending of nonsynchronous amplification factors on machines may provide information on changes in either fluid-induced factors or in shaft anisotropy. The latter, in turn, may be affected by shaft cracking. ■

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